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NUMERICAL STUDY OF TURBULENT FLOW OF PARTLY IONIZED AIR IN A VISCOUS SHOCK LAYER

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An effective method is developed for solving equations for a complete viscous shock layer based on global iterations for the longitudinal component of the pressure gradient and the shape of the leading shock wave. Algebraic models of turbulence for describing transitional and turbulent flow regimes in a chemically nonequilibrium complete viscous shock layer are analyzed. The results obtained are compared with those of actual experiments with entry of bodies into the atmosphere.

Equations for a complete viscous shock layer (CVSL) [1-3] are currently used extensively in order to study supersonic continuous flow past smooth bodies by a viscous gas over a wide range of Reynolds numbers embracing flow from regimes with slip to regimes with formation of both a laminar and turbulent boundary layer at the body. Equations for CVSL follow from Navier–Stokes equations if the latter contain terms of the order O(1) and $O(Re^{-1/2})$ and terms of the order $O(Re^{-1})$ are ignored which are responsible for molecular transfer of mass, pulse, and energy along a coordinate line connected with the main flow direction. As boundary conditions for these equations at the outer boundary (shock wave sought) use is made of generalized Rankine–Hugoniot conditions which take account the effects of molecular transfer in the zone of a jump in thickening with the same asymptotic order of accuracy with respect to Reynolds number Re as for the CVSL equations themselves. Boundary conditions at the body are similar to those for boundary layer equations.

Difficulties in solving CVSL equations by stepping methods along the main flow direction are connected with the fact that in them all of the terms of Euler equations are considered, in particular terms responsible for transfer of perturbations upwards through the flow in subsonic flow regions (longitudinal components of the pressure gradient). For this reason stepping solution methods are incorrect [4]. Additional difficulties arise in solving the problem of supersonic flow past long thin bodies since int his case the shock wave thickens and the thickness of the subsonic region around the body increases. It is noted that with flow past long thin bodies at a sufficient distance from the critical point a turbulent flow regime is possible even with moderate Reynolds numbers in an approach flow.

In this work an effective method is developed for solving equations for a complete viscous shock layer on the basis of global iterations for the longitudinal component of the pressure gradient and the shape of the leading shock wave. The new method is suggested and realized for determining them in each global iteration making it possible to consider the effect of all of the points along the coordinate line on the transfer of perturbations upwards through the flow. Here with a high order of approximation it is not necessary to use special difference equations at the break point for curvature of the body surface as in [3, 5, 6].

Combined solution of all of the equations for a viscous shock layer in contrast to their successive solution in [1] made it possible to improve markedly the stability of the numerical algorithm and thereby to study for long thin bodies over a wide range of dimensionless parameters of the problem. This approach also makes it possible to use a computer with parallel processors which markedly saves computation time.

In studying flow of a dissociated and partly ionized multicomponent mixture with different diffusion properties for the components an algorithm is used which does not require prior resolution of Stefan–Maxwell relationships (equations of component transfer) with respect to diffusion flows. This also reduces the amount of computation since the calculation time becomes proportional to the number of components and not the square of it.

The method suggested makes it possible in a unique way to calculate flow in subsonic and supersonic flow regions, it is much more economic with respect to computation time and the computer memory used compared with standard methods.

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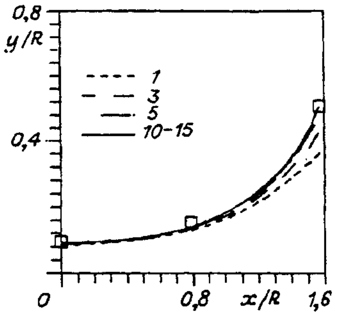

Fig. 1

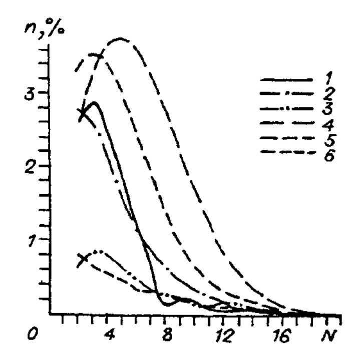

Fig. 2

In order to determine integral characteristics such as heat flow and pressure at the body with an accuracy of 1% not more than two to three global iterations are required. Use of algebraic turbulence models makes it possible to study laminar, transitional, and turbulent flow over the whole range of velocities for occurrence of dissociation and ionization reactions (from frozen to equilibrium).

We consider supersonic flow past the windward surface of an axisymmetric or plane body with continuous or discontinuous curvature of the surface. The set of equations and boundary conditions for the CVSL initially written in a curvilinear orthogonal coordinate system (x, y) naturally connected with the body are written in variables of the Dorodnitsyn type (ξ, η) [2]. In the system obtained physical coordinate y is considered as the function sought $y = y(\xi, \eta)$. The equation for y is a result of equations of state, continuity, and the projection of the equation for pulses of the normal . As boundary conditions for y we use $y = 0$ at the body and a transformed condition for pressure in the shock wave.

In the first global iteration the slope of the shock wave was prescribed analytically [2]:

$$\mathrm{tg}\,\beta_s = \frac{1}{H_1}\frac{dy_s}{dx} = c(\sqrt{(2c-1)/c^2 + \mathrm{tg}^2\alpha} - \mathrm{tg}\,\alpha),\ \beta_s = \beta - \alpha.$$

Here β and α are slopes of the shock wave and body surface to the axis of symmetry; H_1 is Lame coefficient. For function $c = c(\xi)$ a semi-empirical differential equation is suggested

$$\frac{dc}{dx} = \frac{1-k}{2R}c\sqrt{2c-1},$$

and use is made of approximation equations [7] for shock wave departure and curvature at a critical point, conditions of shock wave curvature at the contact point of the sphere–cone, and the tendency of its slope at infinity towards the Mach angle; parameter k lies within the limits from 0.5 to 1 (for a sphere it equals the ratio of shock wave and body curvature with α = 0). The longitudinal component of the pressure gradient in first global iteration was approximated as

$$\frac{\partial p}{\partial \xi} = \omega\frac{p^i - p^{i-1}}{\xi^i - \xi^{i-1}} + (\omega - 1)\left(\frac{\partial p}{\partial \xi}\right)^0,$$

where $\omega = \gamma M^2/[1 + (\gamma - 1)M^{2]}$ in the supersonic flow region [8] and $\omega = 1$ in the supersonic flow region. Here γ is the ratio of specific heat capacities; M is local Mach number; $(\partial p/\partial \xi)^0$ is the initial distribution of $\partial p/\partial \xi$.

In order to calculate new values of $\partial p/\partial \xi$ and $\partial y_s/\partial \xi$ with transition to the next global iteration the distribution of pressure along the coordinate line η = const and departure of the shock wave smoothed by means of minimizing the functional

$$\int_0^{\xi_{max}} [(\varphi - \varphi_0)^2 + \lambda^2\dot{\varphi}^2]d\xi,$$

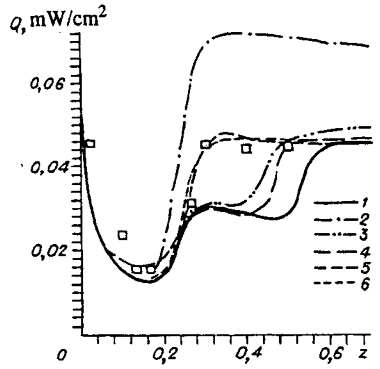

Fig. 3

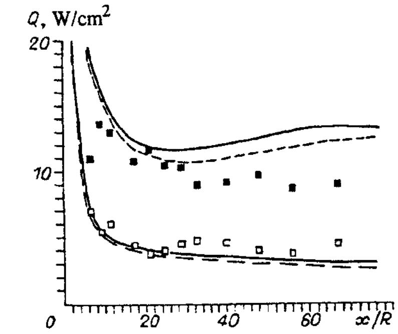

Fig. 4

where φ_0 and φ are any of the functions before and after minimization; a period means derivative with respect to longitudinal coordinate; λ is smoothing parameter [9]. With boundary conditions $\varphi(0) = \varphi_0(0)$, $\varphi(\xi_{max}) = \varphi_0(\xi_{max})$ the extreme problem is reduced to a set of two differential equations of the first order:

$$\lambda^2 \frac{d\dot{\varphi}}{d\xi} = \varphi - \varphi_0, \ \frac{d\varphi}{d\xi} = \dot{\varphi},$$

which was solved a difference method of the fourth order of approximation accuracy [10]. It is essential here smoothing fields φ and $\dot{\varphi}$ are found simultaneously. This method for calculating $\dot{\varphi}$ effectively considers propagation of perturbations upwards through the flow since in new values of $\dot{\varphi}$ there is consideration of the effect of all of the points along each coordinate line η = const. In addition, at the point of body surface curvature discontinuity with a high order of approximation for the shape of the shock wave and the projection of the pressure gradient it is not necessary to use special difference equations as in [3, 5, 6] if one of the calculation points is placed at this point and the change in pitch is considered on coordinate lines ($d\xi = d\xi_0\sqrt{H_1}^2 + (\cos\alpha/R_0\cdot\partial y/\partial\xi)^2$, $d\xi_0$ is pitch along ξ with $\eta = 0$, R_0 is blunting radius).

With prescribed longitudinal components of the pressure gradient and the shape of the shock wave initial with respect to ξ and boundary with respect to η the problem was resolved by a difference method [11] developed previously by the authors for solving nonlinear markedly interconnected sets of equations of the parabolic type. An implicit difference scheme was used of improved approximation accuracy [10], and the Newton method was applied to the nonlinear set of difference equations. A correction system was resolved by trial runs.

Calculations performed for different grids showed demonstrated reliable results.

Given in Fig. 1 are the results of calculating shock wave departure with flow spheres of chemically reacting nine-component partly ionized air (R_0 = 0.635 cm, M_∞ = 15.3, Re_∞ = $1.5\cdot10^4$). Numbers on the curves in Fig. 1 correspond to the numbers of global iterations N. The results obtained are in good agreement with experimental data [12]. The convergence rate for global iterations with respect to different parameters for this calculation is illustrated in Fig. 2. Curve 1 relates to departure of a shock wave, 2 to a derivative with respect to longitudinal coordinate for departure of a shock wave, 3 to pressure at the body surface, 4 to a derivative with respect to longitudinal coordinate for pressure at the body surface, 5 to a derivative with respect to longitudinal coordinate for pressure behind the shock wave, and 6 to convective heat flow towards the body surface. It is noted that in order to establish departure and the shape of a shock wave with an accuracy to n = 1% (mean square) six to seven global iterations are required although the distribution of convective heat flow towards the surface and pressure at it are established after two global iterations. At the same time in order to establish the pressure gradient with respect to the longitudinal coordinate ten to twelve global iterations are necessary.

Presented in Fig. 3 are the results of calculations for some algebraic models of the turbulence of convective heat flows Q towards the side surface of a hyperboloid of rotation modelling spatial flow for the spill lines of Space Shuttle equipment [13]. Here z is coordinate along the axis of symmetry, which is dimensionless for the radius of curvature at the critical point. The conditions in the approach stream correspond to a height of 45.3 km for the actual planned trajectory of entry into the

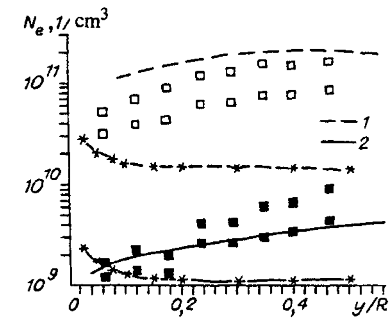

Fig. 5

atmosphere of Earth. Turbulence models of Sebechi–Smith, Lotsyanskii, and Kendall (Curves 2-4 respectively) give in the region of a developed turbulent flow regime heat flows which agree satisfactory with experimental data [13]. In the transition flow region the turbulence models lead to low heat flows which may be explained by high values of viscous underlayer thickness prescribed in these models. However, failure to consider the effect a viscous underlayer in the Escudier model (Curve 1) markedly increases the level of heat flows for all of the flow regimes. Consideration of roughness for the flow surface in the Dam turbulence model (Curve 6) reduces the thickness of the viscous underlayer and makes it possible to obtain satisfactory agreement of heat flow with experimental data [13] in the transition region. The selection of coefficients in the approximation relationship for the thickness of the viscous underlayer in the Sovershennyi turbulence model [14] also describes successfully zones of transitional and developed turbulent flow regimes (Curve 5).

Data presented in Fig. 4 are in good agreement with distributed heat flows obtained along the surface with experimental data [15] and calculations by other authors [15] (dashed lines) in the case of flow past a long thin cone blunted to a sphere ($R_0 = 1.01$ cm, $\alpha = 5.25°$, $M_\infty = 11$, $Re_\infty = 3.3 \cdot 10^5$) with both laminar (open squares) and with turbulent (shaded squares) flow regimes. The turbulence model in [14] refined in [11] is used.

Given in Fig. 5 is comparison of the levels of ionization in the shock wave with flow past a cone blunted to a sphere ($R_0 = 15.4$ cm, $\alpha = 9°$) with the results of measurements provided in [16] for different conditions in an approach flow (curves 1 and 2 are $M_\infty = 25.9$, $Re_\infty = 6.28 \cdot 10^3$ and $M_\infty = 28.9$, $Re_\infty = 1.59 \cdot 10^3$ respectively). The nature of the d distribution obtained for the numerical density of electrons N_e is much better than in calculations with us of the Navier–Stokes parabolized equations [16] (curves labelled with asterisks) where a seven-component model of air was considered and noncatalytic boundary conditions were used charged components.

We note that the method suggested makes it possible to save markedly on the resources of a computer since in the working memory it is only necessary to store the functions sought in two neighboring sections. In addition, several global iterations are required for convergence which is an order of magnitude less than the number of global iterations required in the standard methods. Here the rate of convergence does not depend on the pitch of the grid in the transverse direction.

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